

# Exploiting Time-Varying Relationships in Statistical Relational Models

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# Outline

- 1 Introduction
  - The Basic Problem That We Studied
  - Related Work
- 2 Our Results/Contributions
  - Algorithm
  - Experiments and Results
- 3 Conclusions
  - Summary
  - Future Work



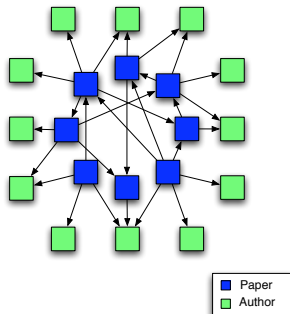
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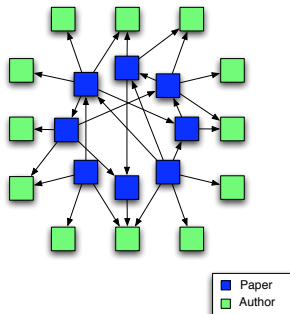
# Example Task

- **Data:** Citation data with papers, authors, references.
- **Task:** Predict paper topic given coauthor and reference information.
- **Current Models:** Relational learning has achieved significant performance gains by exploiting homophily.



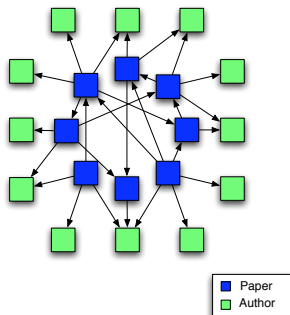
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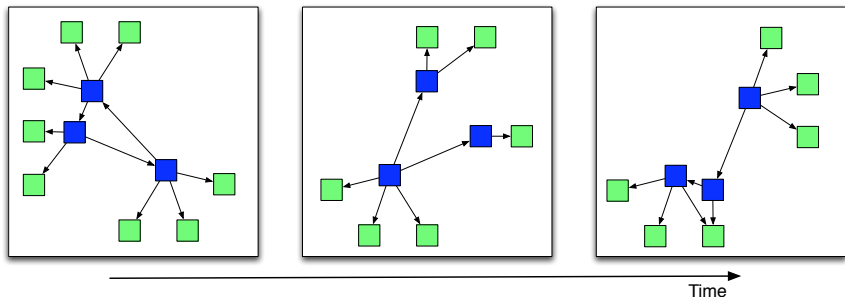


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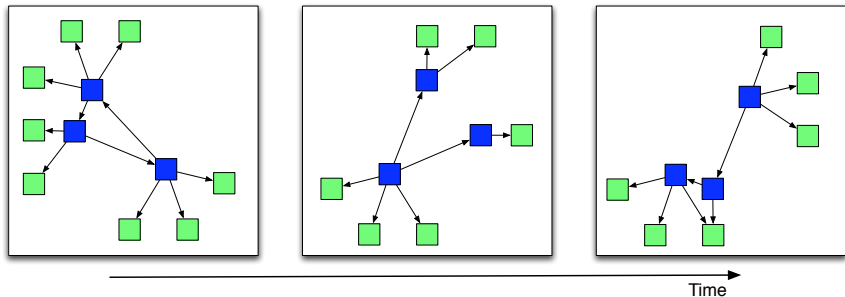
# Why model time?



- Many relational domains have temporal dynamics (e.g. fraud detection, web analysis, bioinformatics, etc).
- Temporal aspects contain information that is important to model (e.g. locality, recurrence).



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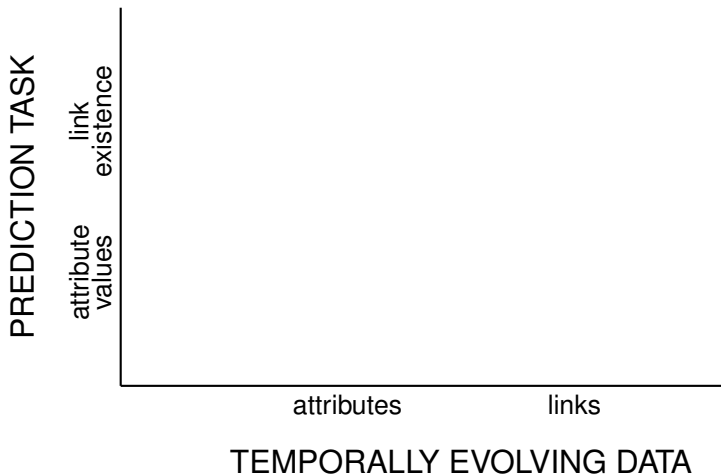


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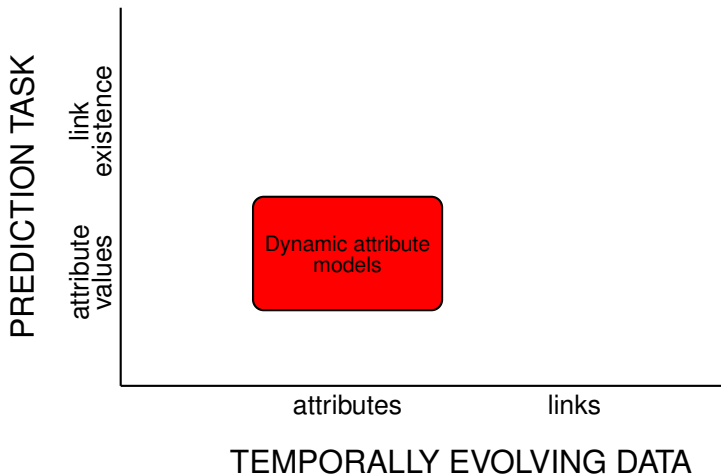


# Temporal Relational Models



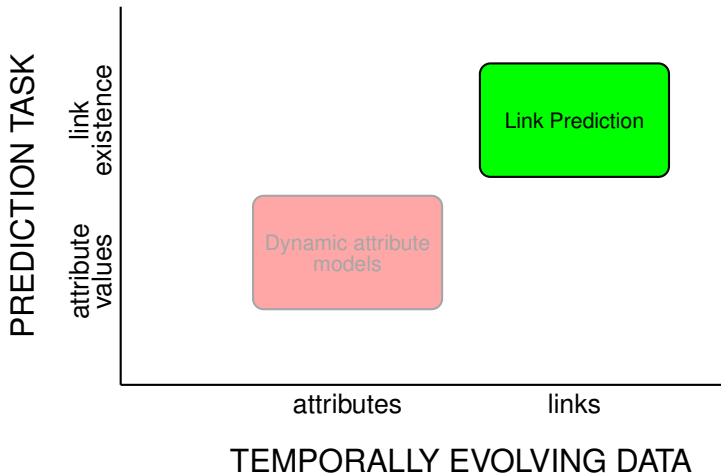
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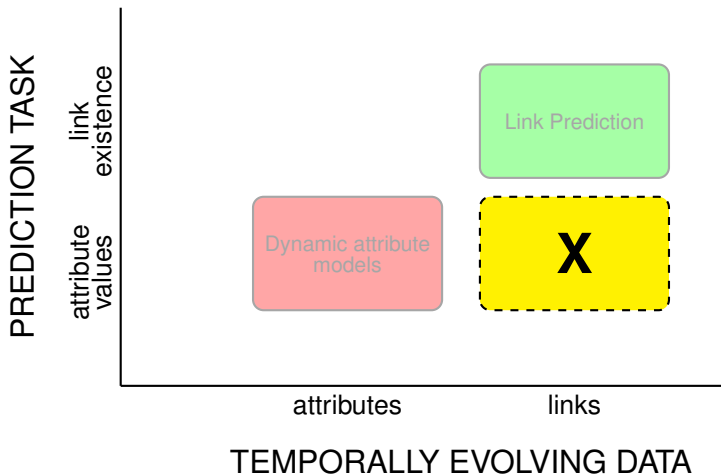


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# Time Varying RBC

- Model domains where link occurrence varies over time.
- Our algorithm takes a two-step approach:
  - Graph Summarization to capture link dynamics.
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# Graph Summarization

- Represent time-varying data as a temporal sequence of graphs ( $G_1, G_2, \dots, G_t$ ).
- Summarize graph  $G_t^S$  as a weighted sum of graphs at each time step:

$$G_t^S = \begin{cases} (1 - \theta)G_{t-1}^S + \theta G_t & \text{if } t > 1 \\ \theta G_t & \text{if } t = 1 \end{cases}$$

- $\theta$  is a weighing parameter defining the strength of a link (i.e. relationship) through time.



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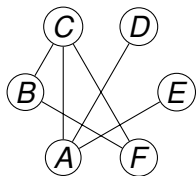
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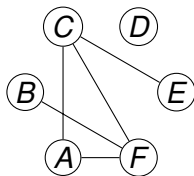
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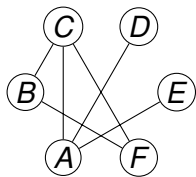


$G_{1991}$

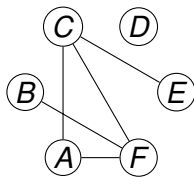


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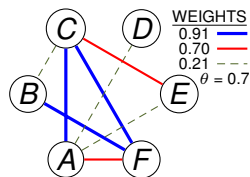
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# Relational Bayes Classifier

## Baseline RBC (Neville et al. '03)

For a class label  $C$ , attributes  $\mathbf{X}$ , and related items  $R$ , the RBC calculates the probability of  $C$  for an item  $i$  of type  $G(i)$  as follows:

$$P(C^i | \mathbf{X}, R) \propto \prod_{X_m \in \mathbf{X}^{G(i)}} P(X_m^i | C) \cdot \prod_{j \in R} \prod_{X_k \in \mathbf{X}^{G(j)}} P(X_k^j | C) \cdot P(C)$$

## TV-RBC

We incorporate the weights from the summary graph (each edge  $(i, j)$  has weight  $w_{ij}$ ) as:

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# Experimental Setup

- **Dataset:** 4,330 CORA machine learning papers published between 1981 and 1998.
- **Task:** Predict paper topic given topics of references and coauthor papers.
- **Models:**
  - *TV-RBC:* learn weighted RBC on  $G_t^S$ , apply on  $G_{t+1}^S$ .
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- **Experiments:**
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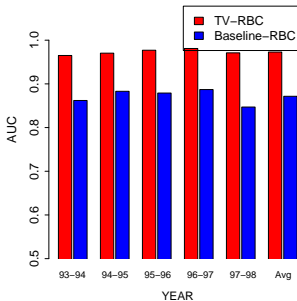
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Variation of AUC across different years for  $\theta = 0.7$



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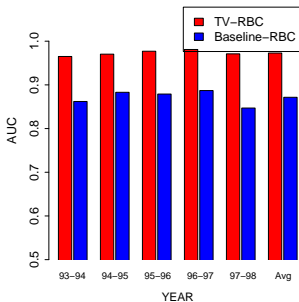
Refs + Coauthors

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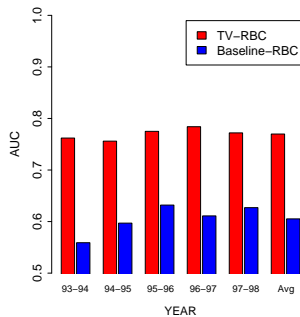


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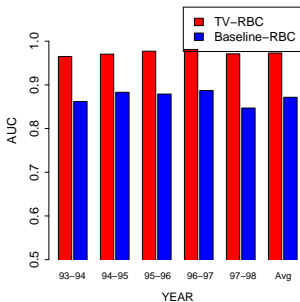
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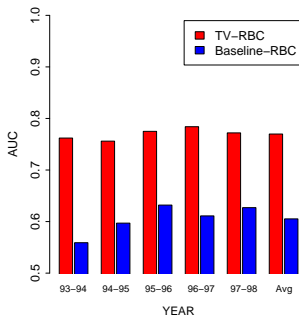


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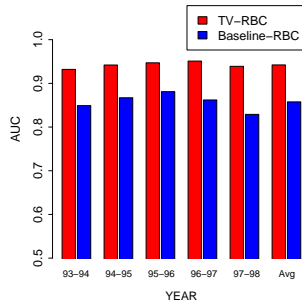
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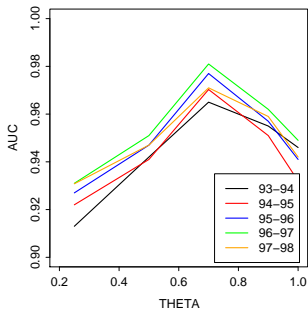
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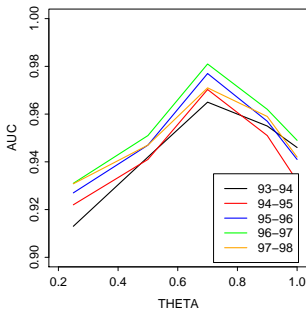
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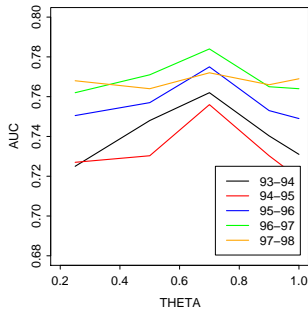


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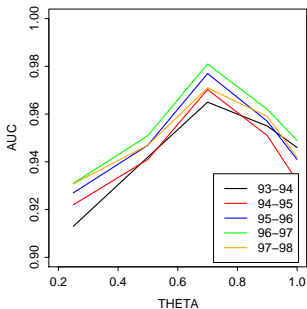
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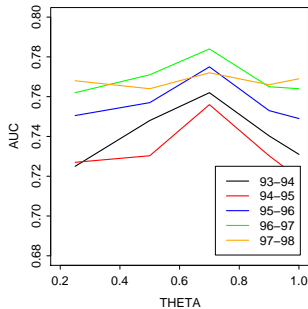
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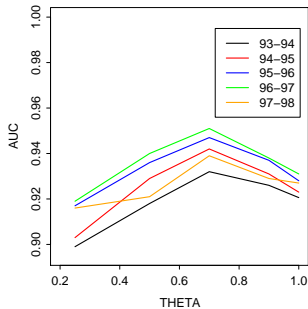
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# Conclusions

- New approach to modeling relational data with time-varying link structure.
- Time-Varying RBC - uses summarized link graph and weighted relational bayes classifier for prediction.
- Our approach to summarization can be used with other SRL models as well (provided attribute values can be weighted).



# Future Work

- Evaluate the algorithm on other real-world datasets.
- Compare the performance of TV-RBC with other modified SRL models (e.g. RPTs).
- Develop a temporal cross-validation approach to set the value of  $\theta$  automatically during learning.
- Extend the approach to model temporally-varying attributes.



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- Prof. Jennifer Neville,  
Department of Computer Science,  
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